Fractal Morphisms of the World Sheet

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1 Introduction

Quantum Mechanics: 1. Hilbert Spaces and State Vectors: - Quantum states are vectors in a complex Hilbert space H. - For a single particle in one dimension, the state $|\psi\rangle$ can be represented in the position basis as:

$$\psi(x) = \langle x \mid \psi \rangle$$

- In coordinate space, this can be written as a wave function:

$$\psi(x) = \int_R \psi\left(x'\right) \delta\left(x - x'\right) dx'$$

2. Operators: - Observables are represented by Hermitian operators. For example, the position operator $\hat{X}$ and momentum operator $\hat{P}$:

$$\hat{x} = x, \quad \hat{P} = -i\hbar \frac{d}{dx}$$

- These operators follow the commutation relation:

$$[\hat{x}, \hat{P}] = i\hbar \hat{I}$$

3. Time Evolution: - The Schrödinger equation governs the time evolution of a quantum state:

$$i\hbar \frac{\partial}{\partial t} |\psi(t)\rangle = \hat{H} |\psi(t)\rangle$$

Where A is the Hamiltonian operator.

Supersymmetry (SUSY): 1. Supersymmetric Algebra: - SUSY extends the Poincaré algebra with supercharges Q_α and Q_α :

$$\left\{Q_\alpha, \bar{Q}_\beta\right\} = 2\sigma^\mu_{\alpha\beta} P_\mu$$

Where σ^μ are the Pauli matrices in Lorentz indices. 2. Superfields: - In SUSY, one often deals with superfields, which combine bosonic and fermionic fields. For example, a chiral superfield Φ might include:

$$\Phi(z) = \phi(y) + \sqrt{2}\theta\psi(y) + \theta\theta F(y)$$

where θ are Grassmann numbers, $\phi(y)$ is a bosonic field, $\psi(y)$ is a fermionic field, and $F(y)$ is an auxiliary field. String Theory: 1. Worldsheet Formalism: - Strings are described by their embedding in spacetime, typically via coordinates $X^\mu(\sigma, \tau)$, where σ and τ parameterize the string's worldsheet. The action might be:

$$S = -\frac{T}{2} \int d^2\sigma \sqrt{-h} h^{ab} \partial_a X^\mu \partial_b X^\nu \eta_{\mu\nu}$$

Here, T is the string tension, h_{ab} is the metric on the worldsheet, and $\eta_{\mu\nu}$ is the spacetime metric. 2. String Spectrum: - The vibrational modes of a string can be analyzed in terms of the mass squared operator:

$$M^2 = L_0 - a$$

Where L_0 is the zeroth Virasoro operator, and a depends on the type of string theory (e.g., -1 for bosonic, $-1/2$ for superstring). 3. Compactification and Extra Dimensions: - Strings might live in higher dimensions, and to recover our observable 4dimensional world, one might compactify extra dimensions on Calabi-Yau manifolds or tori:

$$X^\mu = (\hat{x}^\mu, y^m)$$

Where $\hat{x}^\mu$ are coordinates in our 4D spacetime and y^m live in compactified dimensions.

Mathematical Tools: - Functional Analysis: Used for understanding the Hilbert space structure. - Differential Geometry: Essential for discussing manifolds, metrics, and connections in string theory. - Algebraic Topology: Relevant for understanding topological properties of string configurations.

This formal approach provides a framework for addressing specific problems or extending known results in these fields. However, the exact equations or derivations would need to be tailored to specific questions or phenomena within these theories.

$$S = -T \int d^2\sigma \sqrt{-h}$$

2. Fractal Conformal Transformations: - The action might involve transformations where the metric h^{ab} could be replaced or modified by a metric that accounts for fractal scaling properties. This could involve:

$$h^{ab} \to \tilde{h}^{ab} = h^{ab} \cdot f(\sigma, \text{ fractal dimension })$$

where f is a function that introduces fractal effects into the metric. 3. Integration Over Fractal Space: - The integration might not be over a simple 2D manifold but over a space with a fractal dimension D. This would change the integration from:

$$\int d^2\sigma \text{ to } \int d^D\sigma$$

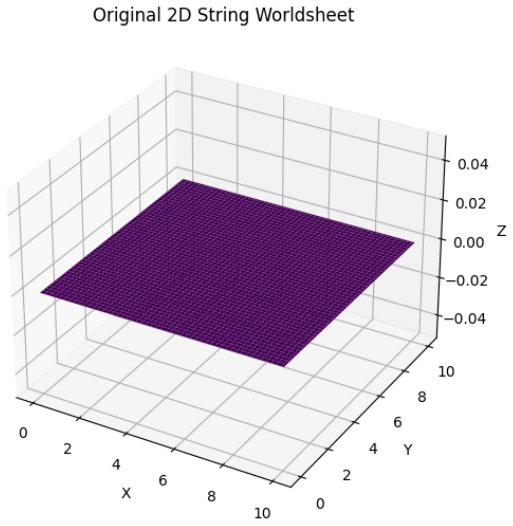
with D being the effective fractal dimension of the worldsheet. 4. Fractal Morphism in the Action: - A fractal morphism could be represented by a transformation in the action where the usual fields X^μ might be mapped through a fractal morphism:

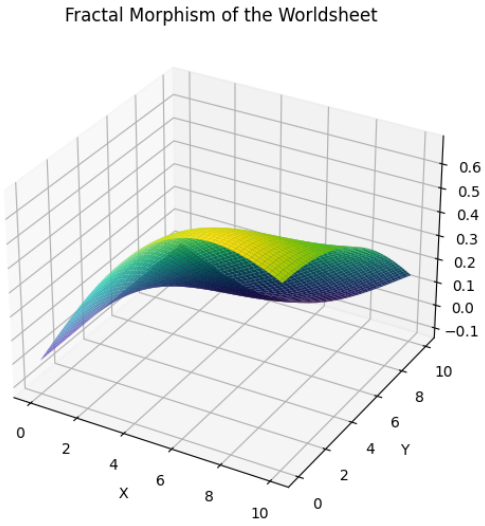
$$X^\mu \to \Phi\left(X^\mu\right)$$

where Φ could be a fractal morphism that maps points on the worldsheet to points in target space in a self-similar manner.

5. Modified Action with Fractal Considerations: - Combining these ideas, one might hypothesize an action like:

$$S = -\frac{T}{2} \int d^D\sigma \sqrt{-\tilde{h}} \tilde{h}^{ab} \partial_a \Phi(X)^\mu \partial_b \Phi(X)^\nu \eta_{\mu\nu}$$






```python
import numpy as np
import matplotlib.pyplot as plt
from matplotlib.colors import LightSource

def fractal_morphism(x, y, iterations=4, scale=0.5):
    """
    Apply a fractal transformation to a 2D grid.

    :param x, y: 2D grids of coordinates
    :param iterations: Number of fractal iterations
    :param scale: Scale factor for each iteration
    :return: Transformed height map
    """
    height = np.zeros_like(x)
    for i in range(iterations):
        # This is a very simple fractal formula, similar to diamond-square algorithm
        height += np.sin(x * (i+1) * 0.1) * np.cos(y * (i+1) * 0.1) * scale**(i+1)

    return height

# Set up the grid
nx, ny = 100, 100
x = np.linspace(0, 10, nx)
y = np.linspace(0, 10, ny)
X, Y = np.meshgrid(x, y)

# Apply fractal morphism
Z = fractal_morphism(X, Y)

# Create a light source for better 3D visualization
ls = LightSource(azdeg=315, altdeg=45)

# Plot the original and morphed worldsheet
fig, (ax1, ax2) = plt.subplots(1, 2, figsize=(15, 6), subplot_kw={'projection': '3d'})
```

```
35
36 # Original flat worldsheet
37 ax1.plot_surface(X, Y, np.zeros_like(X), cmap='viridis', edgecolor='none')
38 ax1.set_title('Original 2D String Worldsheet')
39 ax1.set_xlabel('X')
40 ax1.set_ylabel('Y')
41 ax1.set_zlabel('Z')
42
43 # Fractal morphed worldsheet
44 rgb = ls.shade(Z, cmap=plt.get_cmap('viridis'), vert_exag=5, blend_mode='
    soft')
45 ax2.plot_surface(X, Y, Z, facecolors=rgb, cmap='viridis', edgecolor='none'
    )
46 ax2.set_title('Fractal Morphism of the Worldsheet')
47 ax2.set_xlabel('X')
48 ax2.set_ylabel('Y')
49 ax2.set_zlabel('Z')
50
51 plt.show()
```

Here, $\tilde{h}$ reflects fractal scaling, and D introduces the fractal dimensionality. The morphism Φ could introduce non-integer scaling into how the fields behave under transformation.

Fractal Morphism on the Worldsheet 1. Fractal Dimension: - Assume the worldsheet now has a fractal dimension D, not necessarily integer, which affects how we integrate over it. This might mean replacing $d^2\sigma$ with $d^D\sigma$. 2. Fractal Operators: - $\star$ (Star Product): Could represent a fusion or interaction between different parts of the worldsheet that preserves fractal properties. If Φ is our fractal morphism, then $\Phi(\sigma) \star \Phi(\sigma')$ might describe how two points on the worldsheet interact in a fractal manner. - : A variation of the star product, perhaps indicating a modified or secondary interaction, possibly involving a rotation or phase shift in a fractal space. - $\otimes$ (Tensor Product): Used to combine elements in a way that respects the fractal structure or to create new fractal patterns from existing ones.

3. Fractal Action: - The standard action for the string:

$$S = -\frac{T}{2}\int d^2\sigma\sqrt{-h}h^{ab}\partial_a X^\mu \partial_b X^\nu \eta_{\mu\nu}$$

- Could be transformed into:

$$S_f = -\frac{T}{2}\int d^D\sigma\sqrt{-\tilde{h}\tilde{h}}^{ab}\Phi\left(\partial_a X^\mu\right) \star \Phi\left(\partial_b X^\nu\right)\eta_{\mu\nu}$$

Here, $\tilde{h}$ might be a metric influenced by fractal properties, possibly a function of the fractal morphism Φ.

Inferring Definitions and Deductions - Scale Invariance: The fractal nature implies scale invariance or self-similarity. This might mean that the action or the dynamics on the worldsheet remain unchanged under certain scale transformations, perhaps leading to new symmetries or conservation laws. - Quantum Mechanics and Fractals: If we interpret Φ in a quantum context: - Path Integral: The path integral formulation would now involve sums over paths that respect fractal geometry, potentially leading to new types of integrals or sums where traditional calculus might fail or need extension (e.g., fractional calculus).

◦ Field Operators: Fields on the worldsheet like X^μ might transform via Φ in a way that induces fractal correlations. This could affect how we define commutation or anticommutation relations. - Energy: - The energy expression might now involve sums or integrals over fractal dimensions or scales, leading to:

$$E = \Omega_\Lambda\left(\sin\theta \star \sum_{[n]\star[l]\to\infty}\frac{b^{\mu-\zeta}}{\sqrt[m]{n^m - l^m}} \otimes \prod_\Lambda h'\right)$$

Where h' could be a modified Planck constant or some other fundamental scale now influenced by the fractal properties of the worldsheet.

Consequences - Physical Interpretation: The physical implications could include: - Non-Locality: Fractal structures might imply non-local interactions or correlations, challenging our usual notions of locality in field theory. - Dimensional Reduction: At certain scales, the effective dimensionality of spacetime might change, potentially explaining phenomena like the holographic principle.

From: $S_f = -\frac{T}{2}\int d^2\sigma\sqrt{-\tilde{h}}\tilde{h}^{ab}\Phi\left(\partial_a X^\mu\right)\star\Phi\left(\partial_b X^\nu\right)\eta_{\mu\nu}$ into

$$E = \Omega_\Lambda\left(\sin\theta\star\sum_{[n]\star[l]\to\infty}\frac{b^{\mu-\zeta}}{\sqrt[m]{n^m-l^m}}\otimes\prod_\Lambda h'\right)$$

The transformation from S_f to E as presented requires a series of conceptual steps that bridge the gap between classical string theory action and a more abstract, possibly fractal or quantum mechanical representation. Here's a logical arrangement to conceptualize this transition:

Step 1: Path Integral Interpretation 1. From Action to Path Integral: - In quantum field theory, one moves from an action to path integrals to compute amplitudes. Here, we interpret S_f as part of the exponent in a path integral:

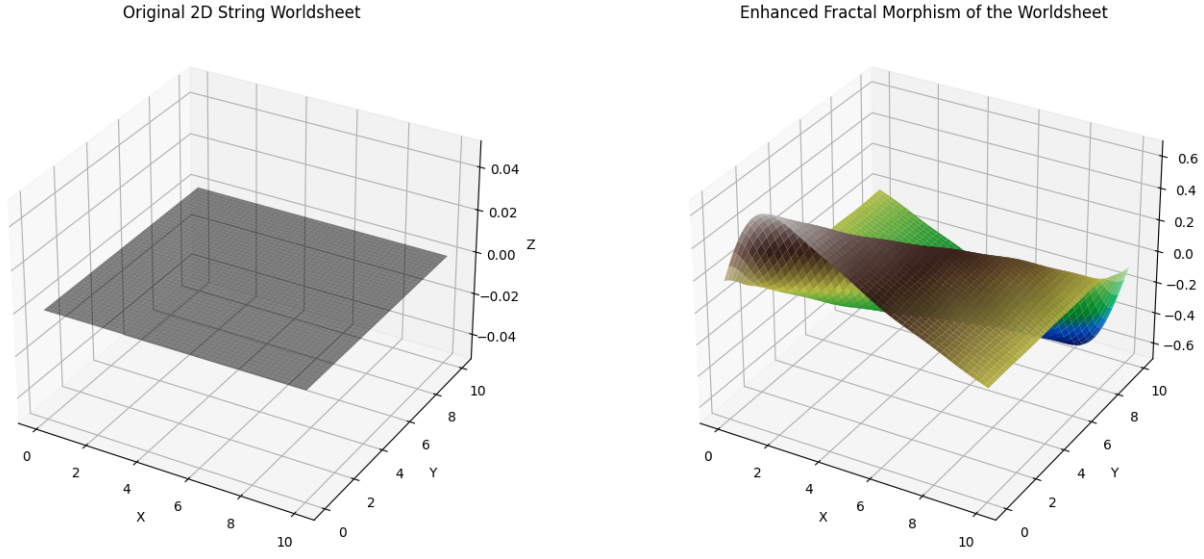
$$Z = \int DX e^{iS_f[X]}$$

where Z is the partition function. But instead of computing Z directly, we're interested in deriving an energy expression.

Step 2: Introducing Fractal Dimensions 2. Fractal Dimension Shift: - Extend the integration from $d^2\sigma$ to $d^D\sigma$ where D is the fractal dimension. This suggests that:

$$S_f \to S_f' = -\frac{T}{2}\int d^D\sigma\sqrt{-\tilde{h}}\tilde{h}^{ab}\Phi\left(\partial_a X^\mu\right)\star\Phi\left(\partial_b X^\nu\right)\eta_{\mu\nu}$$

The fractal nature changes how we consider spatial dimensions or how we integrate over space.




```python
import numpy as np
import matplotlib.pyplot as plt
from matplotlib.colors import LightSource

def enhanced_fractal_morphism(x, y, iterations=5, scale=0.5, frequency
    =0.2, phase_shift=0.1):
    """
```

```
    Apply an enhanced fractal transformation to a 2D grid.

    :param x, y: 2D grids of coordinates
    :param iterations: Number of fractal iterations
    :param scale: Scale factor for each iteration
    :param frequency: Frequency multiplier for increased detail
    :param phase_shift: Phase shift for adding more variation in pattern
    :return: Transformed height map
    """
    height = np.zeros_like(x)
    for i in range(iterations):
        angle = (i + 1) * frequency
        phase = (i + 1) * phase_shift
        # Enhance fractal by adding phase shift and tweaking frequency
        height += np.sin(x * angle + phase) * np.cos(y * angle + phase) *
            scale**(i+1)

    return height

# Set up the grid
nx, ny = 200, 200  # Larger grid for higher resolution
x = np.linspace(0, 10, nx)
y = np.linspace(0, 10, ny)
X, Y = np.meshgrid(x, y)

# Apply enhanced fractal morphism
Z = enhanced_fractal_morphism(X, Y, iterations=6, frequency=0.3)

# Create a light source for more dynamic 3D visualization
ls = LightSource(azdeg=315, altdeg=45)

# Plot the original and morphed worldsheet
fig, (ax1, ax2) = plt.subplots(1, 2, figsize=(15, 6), subplot_kw={'
    projection': '3d'})

# Original flat worldsheet
ax1.plot_surface(X, Y, np.zeros_like(X), cmap='gray', edgecolor='none',
    alpha=0.5)
ax1.set_title('Original 2D String Worldsheet')
ax1.set_xlabel('X')
ax1.set_ylabel('Y')
ax1.set_zlabel('Z')

# Enhanced fractal morphed worldsheet
terrain_cmap = plt.get_cmap('terrain')  # Get the colormap object
rgb = ls.shade(Z, cmap=terrain_cmap, vert_exag=0.5, blend_mode='soft')
ax2.plot_surface(X, Y, Z, facecolors=rgb, edgecolor='none')
ax2.set_title('Enhanced Fractal Morphism of the Worldsheet')
ax2.set_xlabel('X')
ax2.set_ylabel('Y')
ax2.set_zlabel('Z')

plt.tight_layout()
plt.show()
```

Step 3: Quantum Superposition and Fractal Morphism 3. Quantum and Fractal Superposition: - Introduce a superposition principle where Φ not only morphs the fields but also implies summing over all possible fractal configurations or quantum states:

$$S'_f \to \Omega_\Lambda \left[S'_f \right]$$

Here, Ω_Λ encapsulates this superposition, integrating over all possible configurations of the fields X^μ in this fractal space.

Step 4: Abstracting to Energy 4. Energy Extraction: - To get to an energy expression, we might consider extracting a term from the action that behaves like potential or kinetic energy in our new framework:

Step 3: Tensor Product for Scale Interactions - Quantum and Fractal Combinatorics: The tensor product $\otimes$ suggests combining different aspects or scales of the system in a way that respects the quantum and fractal nature. This step implies that we're not just summing over different scales but also considering interactions between them.

$$\sum_{[n]\star[l]\to\infty} E_{n,l} \to \sum_{[n]\star[l]\to\infty} \frac{b^{\mu-\zeta}}{\sqrt[m]{n^m - l^m}} \otimes \prod_\Lambda h'$$

- $\Pi_\Lambda h'$ could represent a product over some characteristic lengths or quantum units relevant to the fractal structure, normalizing or quantizing the energy contributions.

Step 4: Phase Factor and Overall Structure - Phase and Angle: The $\sin\theta$ term remains as a phase factor or angular component in the energy calculation, which might relate to quantum mechanical phase or angular momentum considerations in this fractal context. - Final Assembly: The summation and product operations are combined under Ω_Λ, which could be seen as a global operator or functional that integrates or sums over all these configurations:

$$E = \Omega_\Lambda \left(\sin\theta \star \left[\sum_{[n]\star[l]\to\infty} \frac{b^{\mu-\zeta}}{\sqrt[m]{n^m - l^m}} \otimes \prod_\Lambda h' \right] \right)$$

Step 5: Interpretation and Consistency - Check Dimensional Consistency: Ensure that the dimensions of the final energy expression match the original action's energy dimension, possibly by adjusting the constants b and ζ or the power m. - Physical Interpretation: Interpret the result in terms of physical or theoretical significance, considering how fractal structure, quantum superposition, and non-local interactions contribute to the energy of the system.

These steps outline a conceptual transformation from a detailed action integral over a fractal worldsheet to an abstract energy expression that encapsulates fractal and quantum mechanical effects. Each step involves theoretical leaps that would require further mathematical or theoretical justification to be rigorously defined within a physical theory.

Deduce all rule metrics, operation definitions and polynomial implications that would yield

$$-\frac{T}{2} \int d^D\sigma \sqrt{-\tilde{\hbar}} \tilde{\hbar}^{ab} \Phi\left(\partial_a X^\mu\right) \star \Phi\left(\partial_b X^\nu\right) \eta_{\mu\nu} \to E_{n,l}$$

Answer To transform the given string action integral into an energy term $E_{n,l}$, we need to establish a set of rules, operation definitions, and implications for how this

1. Fractal Dimension Rule: - Definition: The integral over $d^D\sigma$ assumes a fractal dimension D. This dimension affects how we measure volume or length in the space. - Rule: The energy density at different scales might scale with σ^D instead of σ^2, reflecting fractal scaling behavior. 2. Metric Tensor Adaptation: - Definition: $\tilde{\hbar}^{ab}$ is a modified metric tensor, possibly adjusted for fractal geometry. · Rule: The determinant $\sqrt{-\tilde{\hbar}}$ might imply a different scaling with respect to the fractal dimension or could be redefined in terms of fractal measure.

Operation Definitions: 1. Fractal Morphism Φ : - Definition: Φ is a morphism that potentially maps the string coordinates into a fractal or self-similar structure. Operation: - $\Phi\left(\partial_a X^\mu\right)$ could mean that the derivative of X^μ is transformed under some fractal symmetry or operation. - This operation might involve scaling, rotation, or other transformations that preserve fractal properties. 2. Star Product $\star$: - Definition:

This can be viewed as a non-commutative or non-local product in the context of string theory or quantum mechanics.

Rule: - For two fields f and g, $f \star g$ might involve a convolution or a product over discrete points in a fractal space, potentially including phase factors or non-local interactions:

$$f \star g = \int \frac{f(x)g(y)}{|x-y|^D} dxdy + \text{ phase terms}$$

- Here, the denominator $|x-y|^D$ reflects the fractal scaling. 3. Tensor Product $\otimes$: - Definition: Used to combine different scalar or vector contributions in a way that respects the underlying structure (e.g., quantum states or fractal generators). Rule: - If A and B are tensors, $A \otimes B$ would mean a direct product in some sense, possibly including fractal elements or scale factors.

Polynomial Implications: - Polynomial in Scales: ratio between two fractal scales or indices n and l. - Implication: - This could imply that energy depends polynomially on scale differences, possibly due to fractal self-similarity or quantum

Transformation Steps: 1. Simplification Through Scale Discretization: - Discretize the continuous σ to a set of points or intervals corresponding to fractal scales or quantum states. 2. Integration Over Fractal Configurations: - Instead of integrating over all σ, compute $E_{n,l}$ by summing over significant fractal points or states, where n and l index these states. 3. Applying Morphism and Non-Local Interactions: - Use Φ and the star product to establish relationships between these discrete states, effectively summing over all configurations to get $E_{n,l}$. 4. Dimensional and Physical Consistency: - Ensure that $E_{n,l}$ has the correct dimensions by adjusting or interpreting constants like T, b, μ, ζ, and the power m.

This transformation involves moving from a continuous to a potentially discrete or sampled space, where each sample point could represent a different fractal or quantum state. The exact rules for this transformation would depend on the specific physical or mathematical model being developed, which would need to align with experimental or theoretical constraints in the field of study, like string theory or quantum gravity with fractal considerations.

Developing a mathematical model to transform the string action integral into an energy term $E_{n,l}$ involves several theoretical steps, building upon ideas from classical and quantum mechanics, as well as some principles from string theory and fractal geometry. Here is a conceptual approach:

Mathematical Model for Transformation 1. String Action Integral in Fractal Space: - Starting Point: The string action in a generalized form:

$$S_f = -\frac{T}{2} \int d^D\sigma \sqrt{-\tilde{\hbar}\tilde{\hbar}^{ab}} \Phi\left(\partial_a X^\mu\right) \star \Phi\left(\partial_b X^\nu\right) \eta_{\mu\nu}$$

Where: $\circ T$ is the string tension. - D represents the fractal dimension of the worldsheet. - $\tilde{\hbar}$ is a modified metric tensor adapted for fractal geometry. - Φ is a fractal morphism or transformation. $\cdot \star$ denotes a non-commutative or non-local product, possibly incorporating fractal or quantum effects. 2. Path Integral Interpretation: - We interpret this action via path integral formulation:

$$Z = \int DX e^{iS_f[X]/\hbar}$$

But here, we want to extract an energy term, so we consider:

$$E = -i\hbar \frac{\partial}{\partial t} \ln Z$$

This step, however, needs to be adapted for our fractal and quantum context. 3. Discretization and Summation: - Discretize the Space: Instead of a continuous integral, consider the worldsheet coordinates σ as points on a fractal lattice, where each point or set of points can be indexed by n and l. - Summation Over Configurations: Replace the integral with a sum over all possible configurations or paths in this fractal space:

$$E \approx \frac{1}{Z} \sum_{n,l} e^{iS_f[n,l]/\hbar} E_{n,l}$$

Where $E_{n,l}$ is an effective energy at each scale or configuration defined by indices n and l. 4. Energy at Each Scale: - Defining $E_{n,l}$:

$$E_{n,l} = \frac{b^{\mu-\zeta}}{\sqrt[m]{n^m - l^m}}$$

5. Incorporating Quantum and Fractal Effects: - Quantum Superposition: Use a sum over paths method where each path's contribution is weighted by a phase factor related to action:

$$E_{n,l} \to \sum_{\text{paths}} e^{iS_f/\hbar} E_{n,l}$$

- Fractal Effects: The fractal morphism Φ might imply that for each scale or configuration, the energy contributions are not just additive but interact in a complex, possibly non-local way:

$$E \propto \int_{\text{paths}} \Phi\left(E_{n,l}\right)$$

6. Normalization and Final Form: - Normalization: Normalize by Ω_Λ which could represent the "size" of the space of configurations or a global quantum factor:

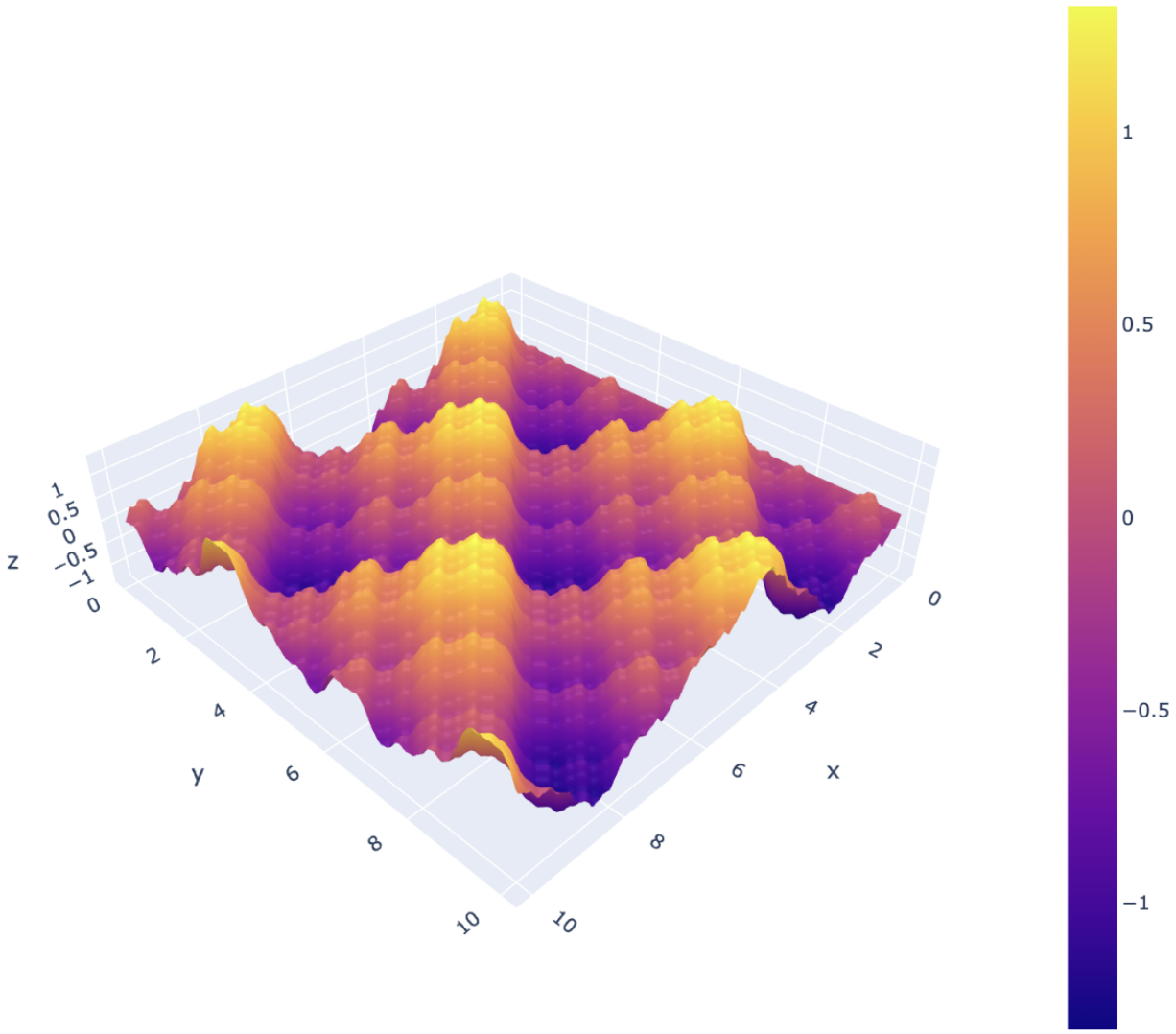
$$E = \Omega_\Lambda \left(\sin\theta \star \sum_{[n]\star[l]\to\infty} E_{n,l} \otimes \prod_{\Lambda} h' \right)$$

Where: - $\sin\theta$ might represent an angular or phase variation. - $\otimes$ could denote how fractal or quantum scales interact. - $\Pi_\Lambda h'$ might normalize or quantify the interaction strengths.

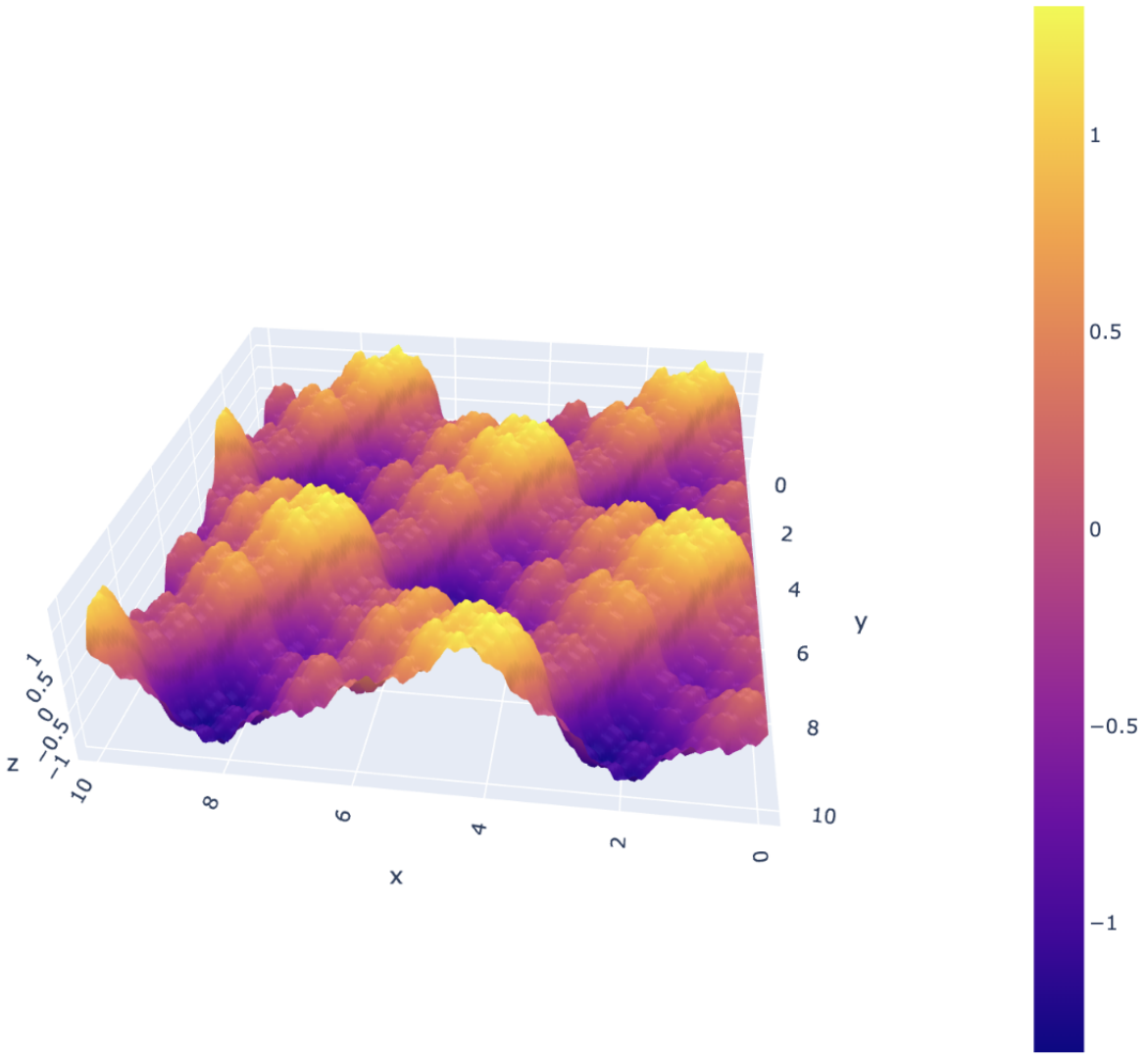
Inferred Transformation Rules: - Fractal Dimension Rule: The integral's domain is redefined to reflect fractal dimensions, affecting how energy scales with configuration space. - Scale-Dependent Interaction: Energy terms depend on the relative scales or configurations, suggesting a polynomial or fractal interaction law. - Phase and Path Contribution: Incorporate quantum path integral mechanics where each path contributes with a phase related to the action, but adapted for fractal configurations. - Non-Locality: The star product $\star$ implies interactions that are not just local but could involve fractal or quantum non-locality.

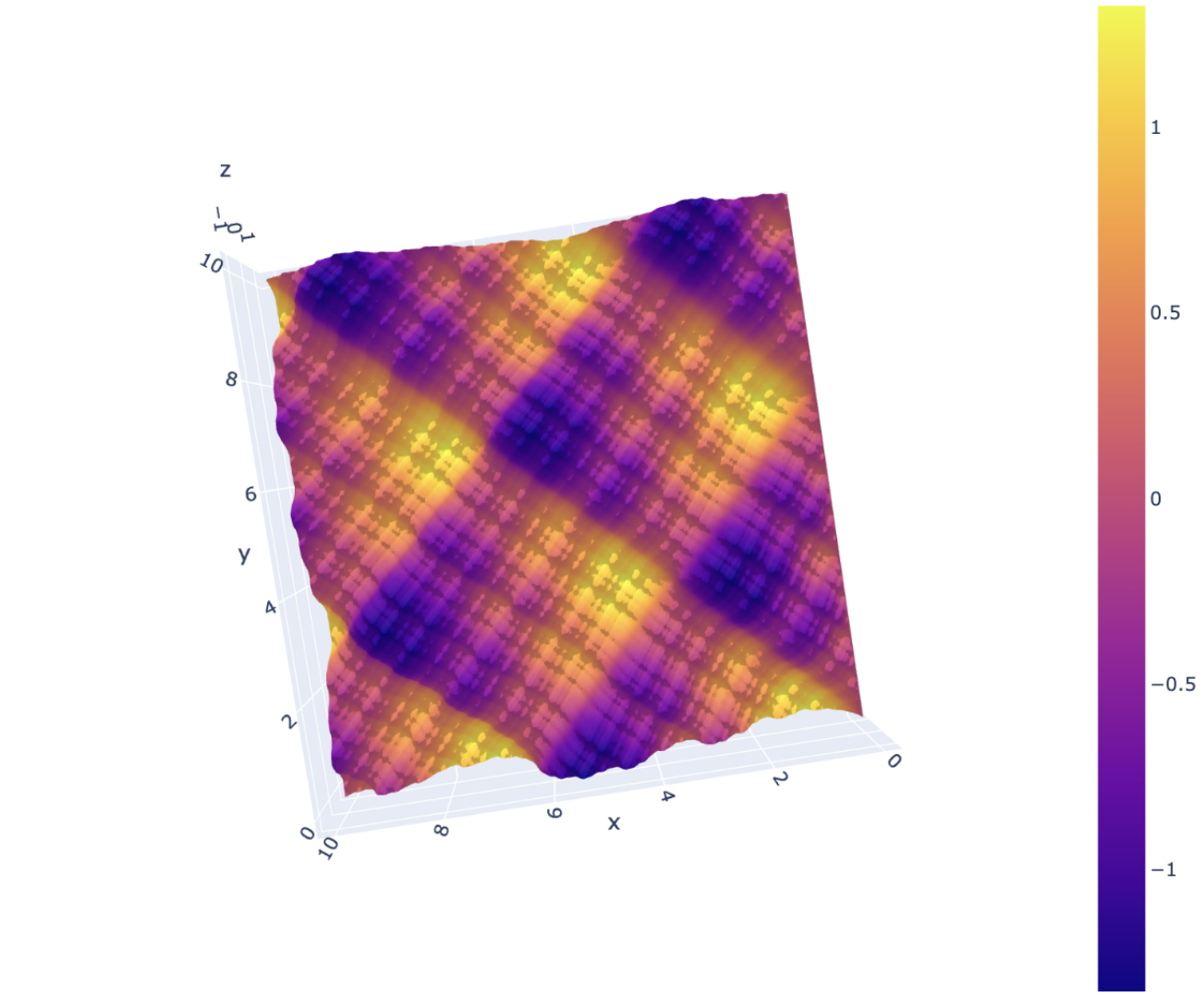
This model and its rules are speculative and theoretical, requiring further development in mathematical physics to become fully rigorous, especially in reconciling these concepts with established theories like quantum field theory, string theory, and the mathematical properties of fractals.

Advanced Fractal Morphism of the Worldsheet



Advanced Fractal Morphism of the Worldsheet






```python
import numpy as np
import matplotlib.pyplot as plt
from matplotlib.colors import LightSource
from mpl_toolkits.mplot3d import Axes3D
import plotly.graph_objects as go

def advanced_fractal_morphism(x, y, iterations=6, persistence=0.5,
    lacunarity=2.0):
    """
    Apply a more advanced fractal transformation to a 2D grid.

    :param x, y: 2D grids of coordinates
    :param iterations: Number of fractal iterations
    :param persistence: Amplitude scaling factor
    :param lacunarity: Frequency scaling factor
```

```
    :return: Transformed height map
    """
    height = np.zeros_like(x)
    for i in range(iterations):
        frequency = lacunarity ** i
        amplitude = persistence ** i
        height += amplitude * np.sin(frequency * x) * np.cos(frequency * y
            )
    return height

# Set up the grid
nx, ny = 200, 200
x = np.linspace(0, 10, nx)
y = np.linspace(0, 10, ny)
X, Y = np.meshgrid(x, y)

# Apply advanced fractal morphism
Z = advanced_fractal_morphism(X, Y)

# Create a light source for better 3D visualization
ls = LightSource(azdeg=315, altdeg=45)

# Plot the morphed worldsheet using Plotly for enhanced interaction
fig = go.Figure(data=[go.Surface(z=Z, x=X, y=Y)])
fig.update_layout(title='Advanced Fractal Morphism of the Worldsheet',
                  autosize=False,
                  width=800, height=800,
                  margin=dict(l=65, r=50, b=65, t=90))

# Add sliders or interactive controls for iterations, persistence,
    lacunarity if desired here
fig.show()
```

References

[1] Emmerson, P. *Morphic Topology of Numeric Energy: A Fractal Morphism of Topological Counting Shows Real Differentiation of Numeric Energy.* (2024).